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August 18, 2015

Origin of Mass and Strong Coupling Gauge Theories
(SCGT15)

Nagoya, Japan

March 3, 2015 through March 6, 2015

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Lattice study of the scalar and baryon spectra in many flavor QCD

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In the search for a composite Higgs boson in walking technicolor models, many flavor QCD, in particular with $N_f = 8$, is an attractive candidate, and has been found to have a composite flavor-singlet scalar as light as the pion. Based on lattice simulations of this theory with the HISQ action, we will present our preliminary results on the scalar decay constant using the fermionic bilinear operator, and the mass of the baryon state which could be a candidate for the dark matter. Combining these two results, implications for the dark matter direct detection is also discussed.

Keywords: Lattice gauge theory, Conformal/Walking Dynamics, Dark matter, composite Higgs particle.

1. Introduction

A new scalar particle with mass 125GeV has been discovered at the LHC, and the current experimental data is consistent with the standard model (SM) of particle physics. However, the origin of the mass and electroweak symmetry breaking remains unknown, and the SM does not have a viable dark matter (DM) candidate, which is one of reasons why we are seeking new physics model beyond the SM.

There is a possibility of a strong gauge dynamics beyond TeV, and one of popular candidates among the strong dynamics is the walking technicolor model to break the electroweak symmetry breaking dynamically in an approximately scale-invariant dynamics, which predicts a large anomalous dimension $\gamma_m \simeq 1$ and a “techni-dilaton” as a pseudo Nambu-Goldstone boson of the spontaneously broken approximate scale symmetry.¹ The Higgs scalar could be identified with the techni-dilaton, which is a composite bound-state of the flavor-singlet techni-fermion bilinear, parametrically

lighter than other techni-hadrons. From the phenomenological point of view, techni-dilaton decay constant (denoted here as F_σ) is a very important parameter as well as the mass, since all the techni-dilaton couplings to the SM particle are controlled by the parameter F_σ . In addition, the technicolor model has a rich hadron structure, e.g. the techni-rho meson which may be discovered in the LHC. Furthermore the techni-baryon, its lightest neutral component, could be a stable particle due to the techni-fermion number conservation. Thus it would be a candidate for the DM.

In order to elucidate the walking technicolor model, fully non-perturbative understanding of the strong gauge dynamics is necessary, and the lattice numerical calculation is the most powerful tool for that purpose. In the previous study in the LatKMI collaboration, it is shown that the 8-flavor ($N_f = 8$) QCD could be a candidate for the walking gauge theory, where we found an approximate hyperscaling in various hadron mass spectra in a certain fermion mass range². Remarkably, the flavor singlet-scalar (σ) mass is as light as pseudoscalar (π), which suggests that σ could be regarded as the techni-dilaton³.

In this proceedings we investigate the decay constant of the flavor-singlet scalar as well as the mass. Using the Ward-Takahashi (WT) identity of the scale symmetry in the continuum theory, we estimate the dilaton decay constant F_σ . As for the techni-baryon DM, it is important to investigate its direct detection, where the most dominant contribution of the DM scattering amplitude is the scalar mediated spin-independent interaction. Based on the DM effective theory whose parameters include F_σ and techni-baryon mass, we evaluate the leading order of the scattering cross section of the DM. Using the lattice input of the above parameters, we discuss experimental detectability and a possibility of techni-baryon DM scenario.

In the next section, we briefly explain how to calculate the mass of the flavor-singlet scalar and scalar decay constant on the lattice. We discuss a relation between scalar and dilaton decay constants. In Section 3, we show our lattice result of the scalar mass and decay constants. The chiral extrapolation can be made using a dilaton effective theory. In Section 4, we study the DM detection rate based on the dilaton chiral perturbation theory with baryon, and the summary is given in Section 5. Note that all the results shown in the proceedings are very preliminary.

2. Scalar mass and decay constant

We investigate the mass of the scalar and its decay constant on the lattice. The mass of the flavor-singlet scalar, which we call σ , is calculated from the two-point correlation function ($C_\sigma(t)$) of the flavor-singlet scalar bilinear operator \mathcal{O}_s . By using the staggered fermion field χ_i on the lattice, \mathcal{O}_s is written as $\mathcal{O}_s(x, t) = \sum_i \bar{\chi}_i(x, t)\chi_i(x, t)$, where i denotes the different staggered fermion species, namely $i = 1, 2$ for $N_f = 8$ QCD. The correlator is then given by

$$C_\sigma(t) = \frac{1}{V} \langle \sum_x \mathcal{O}_s(x, t) | \mathcal{O}_s(0, 0) \rangle, \quad (1)$$

where V is the spacial volume ($V = L^3$). The asymptotic behavior of $C_\sigma(t)$ is given by $C_\sigma(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{SC}}}(t)$, where $A_{\pi_{\overline{SC}}}$ is a pseudoscalar correlator which is a parity-partner of the scalar, and $A_H(t) = A_H(e^{-m_H t} + e^{-m_H(T-t)})$.

We define the scalar decay constant F_S as the scalar operator matrix element,

$$\langle 0 | m_f O_s(0, 0) | \sigma(0) \rangle = F_S m_\sigma^2. \quad (2)$$

The above matrix element can be calculated from $C_\sigma(t)$. Substituting the complete set, $\sum_n |n\rangle\langle n| = \int \frac{d^3 p}{(2\pi)^3} \frac{|\sigma(p)\rangle\langle\sigma(p)|}{2E_p} + \dots$ into Eq. 1, the correlator $C_\sigma(t)$ is given by

$$C_\sigma(t) = \frac{1}{V} |\langle 0 | O_s(0, 0) | \sigma(0) \rangle|^2 \frac{e^{-m_\sigma t}}{2m_\sigma}, \quad (3)$$

Thus we calculate F_S as follows,

$$F_S = \frac{m_f \sqrt{2m_\sigma V A_\sigma}}{m_\sigma^2}. \quad (4)$$

We note that this quantity is renormalization group invariant and a physical quantity, and it is easy to be measured on the lattice. It is also noted that F_S should obey a hyperscaling relation, $F_S \propto m_f^{1/(1+\gamma)}$ with $\gamma \sim \gamma_m$, if the theory is in the conformal window⁴.

Let us now discuss the dilaton decay constant F_σ . The following discussion is based on the continuum theory. It is important to recall the basic definition of the dilaton decay constant, which is $\langle 0 | \mathcal{D}^\mu(x) | \sigma(p) \rangle = -i F_\sigma p^\mu e^{-ipx}$. From this, we obtain $\langle 0 | \partial_\mu \mathcal{D}^\mu(0) | \sigma(0) \rangle = -F_\sigma m_\sigma^2$. Therefore the dilaton decay constant can be directly calculated from the matrix element of the dilatation current. However the dilatation current is rather difficult to construct on the lattice, since it requires a subtraction of the power divergence of the operator.

Instead, we consider an alternative way to estimate it, which can be derived by using the Ward-Takahashi(WT)-identity of the dilatation current. Following the argument in⁵, we consider the integrated WT-identity for dilatation transformation. We use the scale transformation relation of an operator \mathcal{O} , $\delta_D \mathcal{O} = \Delta_{\mathcal{O}} \mathcal{O}$, where $\Delta_{\mathcal{O}}$ is the scaling dimension of \mathcal{O} . In the zero momentum transfer limit WT-identity leads $\int d^4 x T \langle \partial_\mu \mathcal{D}^\mu(x) \mathcal{O}(0) \rangle = \Delta_{\mathcal{O}} \langle \mathcal{O} \rangle^a$, from which, by assuming the σ pole dominance, we obtain $\Delta_{\mathcal{O}} \langle \mathcal{O} \rangle = (-F_\sigma m_\sigma^2) \frac{1}{m_\sigma^2} \langle \sigma | \mathcal{O} | 0 \rangle = -F_\sigma \langle \sigma | \mathcal{O} | 0 \rangle$.^b In the case we take $\mathcal{O}(x) = m_f \sum_i^{N_F} \bar{\psi}_i \psi_i(x)$ (the flavor-singlet scalar operator), we obtain a relation between F_S and F_σ as

$$F_S F_\sigma m_\sigma^2 = -\Delta_{\bar{\psi}\psi} m_f \langle \sum_i^{N_F} \bar{\psi}_i \psi_i \rangle. \quad (5)$$

(Dividing both sides by m_f leads to the relation obtained at $m_f = 0$ ⁶.)

^aIn the formula, the vacuum contribution on the left hand side is absent in the WT identity.

^bSo-called Partially Conserved Dilatation Current (PCDC) relation $F_\sigma^2 m_\sigma^2 = -4 \langle \theta_\mu^\mu \rangle$ follows for $\mathcal{O} = \partial_\mu \mathcal{D}^\mu = \theta_\mu^\mu$ (nonperturbative trace anomaly) with $d_{\theta_\mu^\mu} = 4$.

We note that this relation holds in the continuum theory with infrared conformality under the assumption of the σ pole dominance. While we do not know the real infrared behavior in $N_f = 8$ QCD towards the chiral limit, our recent lattice study shows that an approximate hyperscaling (walking behavior) is found for various hadron spectra in a certain fermion mass region, so the above relation could be effective also in the walking region up to excited state contaminations and discretization effects. Therefore in the following lattice analysis, we shall use this relation for a semi-direct estimate of F_σ . In order to precisely calculate F_σ , a direct calculation for the dilatation current is needed.

3. Result

In this section we show our lattice result in $N_f = 8$ QCD. Details of the simulation and analysis for measurements can be found in Refs.⁸. The results of m_σ and F_S are summarized in Fig 1. It is found that the scalar is as light as π , and clearly lighter than ρ , and we obtain a good signal for F_S as well. The lightness of the scalar could be regarded as a reflection of a dilatonic nature of the scalar.

While our mass is far from the chiral limit, we shall study the chiral extrapolation based on an effective theory, which is dilaton chiral perturbation theory(DChPT)⁷. At the leading of the DChPT, the scalar mass is given by $m_\sigma^2 = d_0 + d_1 m_\pi^2$, where $d_1 = \frac{(3-\gamma_m)(1+\gamma_m)}{4} \frac{N_f F_\pi^2}{F_\sigma^2}$, m_π and F_π are the mass and decay constant of π . We also try a fit with a naive form of $m_\sigma = c_0 + c_1 m_f$. The fit results are shown in the left panel of Fig. 2, where a reasonable value of $\chi^2/\text{dof} \sim \mathcal{O}(1)$ is obtained. The value of m_σ^2 in the chiral limit is $d_0 = -0.0028(98)$. Thus we obtain a very light σ . From the fit value of the slope $d_1 = 0.89(26)$ in DChPT, we also obtain $F_\sigma \sim \sqrt{N_f} F_\pi \sim 0.06$, with F_π being in the chiral limit⁸ and $\gamma \sim \gamma_m \sim 1$.

On the right panel of Fig.2, the result of F_σ from the semi-direct estimate (Eq.5) is shown^c. We also carry out the chiral extrapolation fits. The fit results are also shown in the figure. In the chiral limit, we obtain $\frac{F_\sigma}{\Delta_{\bar{\psi}\psi}} \sim 0.03$. From the fact $\Delta_{\bar{\psi}\psi} = 3 - \gamma$, above two different methods of DChPT and semi-direct calculation give a consistent result of F_σ .

4. Dilaton ChPT with Baryon and Dark matter

In the technicolor model, it may have a good candidate for the composite DM as a neutral bound state made of constituent (possibly charged) techni-fermions. Then the coupling between the SM particle and the DM as well as the mass of the DM are constrained by the experiment of the direct detection of the DM. In most of cases, it observes the scattering rate of the DM which uses a heavy nucleus as a detector. One of dominant contributions is the Higgs (scalar) mediated spin-independent process. In the followings, we provide a low-energy effective theory of the walking gauge theory including the DM, whose low-energy constants can be determined from the lattice results of the baryon and scalar spectra. The result

^cFor the chiral condensate, we use its chiral limit value to avoid large lattice artefacts.

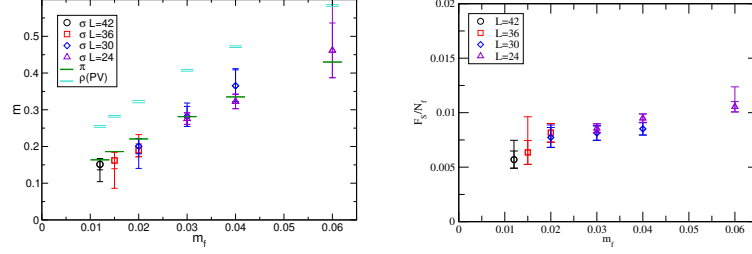


Fig. 1. (Left) Mass of the flavor-singlet scalar. Other hadron masses are also shown. Outer error represents the statistical and systematic uncertainties added in quadrature, while inner error is only statistical. (Right) Scalar decay constant of the flavor-singlet scalar.

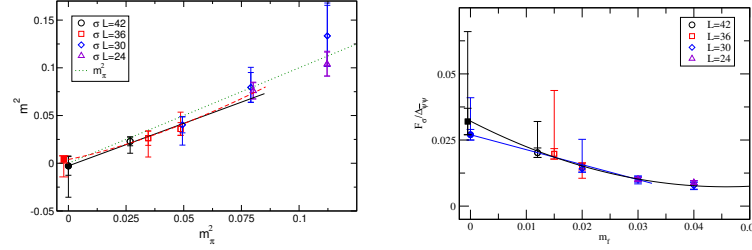


Fig. 2. (Left) m_σ^2 v.s. m_π^2 . Black line shows the fit with DChPT formula. Dashed red curve shows the fit result with a naive form, $m_\sigma = c_0 + c_1 m_f$. (Right) Blue and Black lines show the chiral fits of the linear and quadratic polynomial in m_f with the lightest 4 and 5 data points used. Outer error represents the statistical and systematic uncertainties added in quadrature, while inner error is only statistical.

obtained in the previous section is useful, and it gives rise to information on the DM direct detection experiment.

We consider a DM effective theory including the dilaton based on the DChPT. Since the DM is the (lightest) techni-baryon, an extension to the baryon sector of the DChPT is straightforward as the same manner in⁷. As a result, in the leading order the dilaton field can only couple through the baryon mass term as

$$\mathcal{L} = \bar{B}(x)(i\gamma_\mu \partial^\mu - \chi(x)m_B)B(x), \quad (6)$$

where $\chi(x) = e^{\sigma(x)/F_\sigma}$, $B(x)$ is the baryon field, and m_B is its mass in the chiral limit. The parameter m_B explicitly breaks the scale symmetry, and the (pseudo) dilaton acts on this term to make the action scale invariant. Actually this action is invariant under the scale transformation, $\delta B = (\frac{3}{2} + x_\nu \partial^\nu)B$, $\delta \chi = (1 + x_\nu \partial^\nu)\chi$.

From this effective theory, we can read off the dilaton-baryon effective coupling ($y_{\bar{B}B\sigma}$), which is uniquely determined as $y_{\bar{B}B\sigma} = m_B/F_\sigma$. Regarding to the SM sector, the dilaton-nucleon effective coupling is also determined within the framework of the dilaton effective theory¹⁰, since the dilaton-quark coupling ($y_{\sigma\bar{f}f}$) could be related to the SM Yukawa coupling ($y_{h_{SM}\bar{f}f}$) as $\frac{y_{\sigma\bar{f}f}}{y_{h_{SM}\bar{f}f}} = \frac{(3-\gamma)v_{EW}}{F_\sigma}$. Combined

both SM and technicolor sectors, the cross section with a target nucleus N is given as $\sigma_{SI} = \frac{M_R^2}{\pi} (Z f_p + (A - Z) f_n)^2$, where $M_R(m_B, m_N) = (m_B + m_N)/(m_B m_N)$, and Z and A are the total number of the proton(p) and nucleon(n) in the nucleus. The parameter $f_{(n,p)}$ is defined as $f_{(n,p)} = \frac{m_N}{\sqrt{2} m_\sigma^2} \frac{y_{B B \sigma}}{F_\sigma} (3 - \gamma) \left(\sum_{q=u,d,s} f_{T_q}^{(n,p)} + \frac{2}{9} f_{T_G}^{(n,p)} \right)$, where $f_{T_q}^{(n,p)}$ is a nucleon matrix element of the light quarks ($q = u, d, s$), and $f_{T_G}^{(n,p)}$ is the one of the heavy quarks^d. Thus lattice calculations are used in the technicolor as well as QCD for the DM physics.

Here we show our numerical results of the DM cross section.^e We use the lattice results of the dilaton decay constant (F_σ) obtained from the previous section and baryon mass, while the scalar mass m_σ is fixed to its experimental value (125 GeV) in this analysis. We use the values in Ref.⁹ for $f_{T_q}^{(n,p)}$. As for the scale setting, we use a relation $\sqrt{N_f/2} F_\pi / \sqrt{2} = 246 [\text{GeV}]$ as a sample input. We again use the F_π in the chiral limit. To compare it with the experiment, we use the cross section per nucleon (σ_0) instead of σ_{SI} . The result is shown in Fig. 3. According to the recent DM experiment, our result suggest that it may be difficult to explain an existence of the DM by a techni-baryon. However we note that there exist other contributions to the DM cross section, e.g. gauge boson mediated interaction, and higher order terms, which might affect on the DM cross section. Calculation of these contributions on the lattice is a future work.

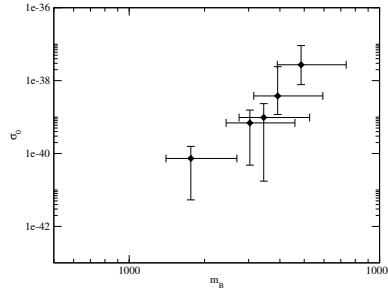


Fig. 3. $\sigma_0 [\text{cm}^2]$ as a function of $m_B [\text{GeV}]$. The results for $m_f = 0.030, 0.020, 0.015, 0.012$, and the chiral limit are shown from up-right to bottom-left. In plots, both the statistical and systematic errors coming from are included.

5. Summary

The scalar mass and decay constant are very important parameters to probe a technicolor signature at the LHC. Based on the lattice theory, we derived a relation

^dIn general, techni-fermion can be charged under the SM color, so there may exist additional contributions to the nucleon matrix elements from the techni-fermions. In this analysis, we omit these contributions for simplicity.

^eThe similar analysis on the lattice has been made for a different composite DM model based on the strong dynamics¹¹.

between scalar decay constant and the (flavor-singlet) scalar correlation functions. Our numerical result shows that the signal of the decay constant is as good as the one of the mass. Though an accuracy of our result is not enough to precisely extrapolate towards the chiral limit, we obtained a rough estimate of the ratio $F_\sigma/F_\pi \sim 3$. It will be useful for the collider phenomenology. Besides it, F_σ can also apply to the DM physics. We provided a DM effective theory based on the dilaton ChPT, where a dilaton-DM coupling is related to F_σ and the DM mass. From the experiment of the DM direct detection, these parameters are constrained. We then discussed a possibility of the techni-baryon DM. We should note that all the results shown here are preliminary. Simulation at lighter fermion mass, and the direct computation on the lattice are needed to precisely estimate the above quantities.

Acknowledgments – Numerical computations have been carried out on the high-performance computing systems at KMI(φ), at the Information Technology Center in Nagoya University (CX400), and at the Research Institute for Information Technology in Kyushu University (CX400 and HA8000). This work is supported by the JSPS Grant-in-Aid for Scientific Research (S) No.22224003, (C) No.23540300 (K.Y.), for Young Scientists (B) No.25800139 (H.O.) and No.25800138 (T.Y.), and also by the MEXT Grants-in-Aid for Scientific Research on Innovative Areas No.23105708 (T.Y.). E.R. acknowledges the support of the U.S. Department of Energy under Contract DE-AC52-07NA27344 (LLNL).

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